SOLVER for Optimization

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Optimization

- The procedures for finding an alternative with the best achievable performance as possible under the given constraints
- Minimization/Maximization
- Optimization tools?

Lingo	CPLEX
Lindo	Mathlab
Excel - Solver	Etc.

Excel Solver

- Solver is numerical optimization add-in software embedded in Excel, but not enable by default
- Used to solve problem to find optimal or near optimal value
- Solve linear/some non-linear problems

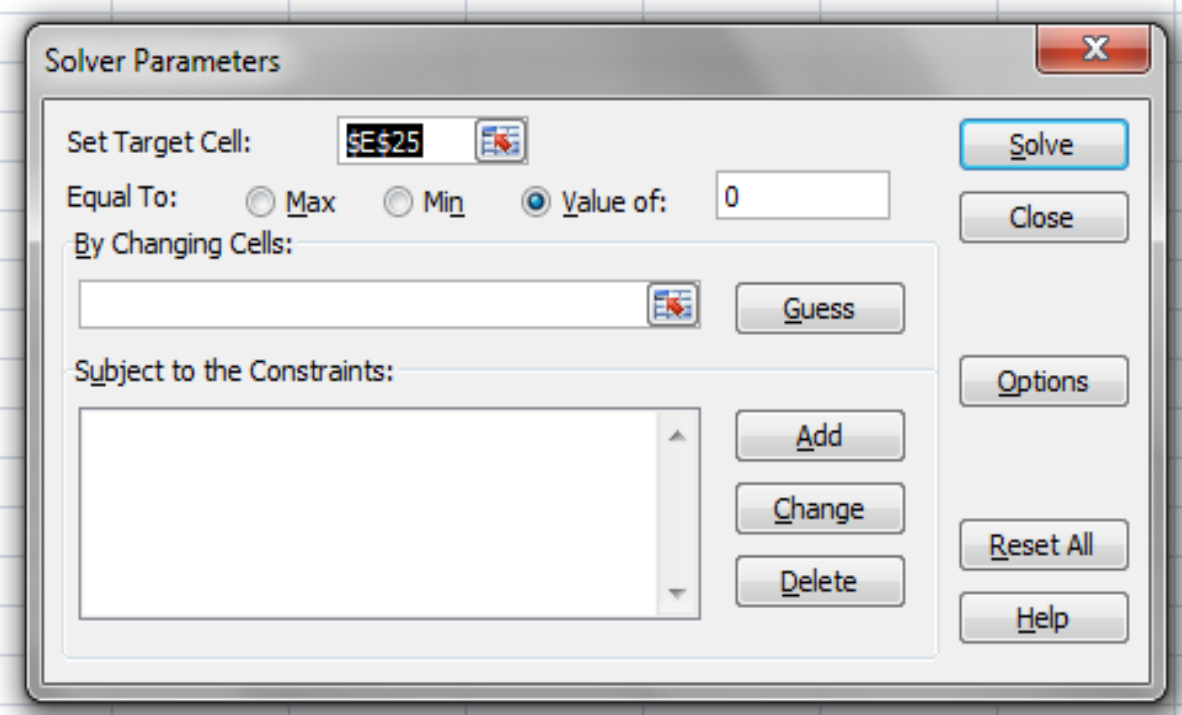
Excel Solver

- Advantages
  - Everything of interests can be tracked (costs, profits, resources availability, resource usage)
  - Cells of interests can be varies
  - Used as “what-if” analysis tool
- Disadvantages
  - Black box
  - Need carefully modeling: constraints

Excel Solver

Main Components :

- ❖Target cells
- ❖Changing cells
- ❖Constraints



Excel Solver

- *Target cell(s)*: contains objective function, the cell that represents the ***goal or objective*** of the problem
- Setting: maximization/minimization/fixed value

	Target Cell
Maximize	Profit
Minimize	Cost
Value of	Net future value

Excel Solver

- *Changing cells /Adjustable cells*: cells that contain ***decision variables***, the cells that can be modified to arrive at the desired outcome, cells that we can change or adjust to optimize the target cell

Problem	Changing cells
Product mix	Amount of each product produced
Product shipping	Shipping quantity
Investment	Money invested in each project

Excel Solver

- *Constraints*:
  - Restrictions or limitations to what Solver can do to solve the problem
  - Restrictions to the changing cells or other cells related
  - The rules which define the limits of the possible solutions to the problem
  - Requirement constraints: given by problem
  - Non-negative constraints

Excel Solver

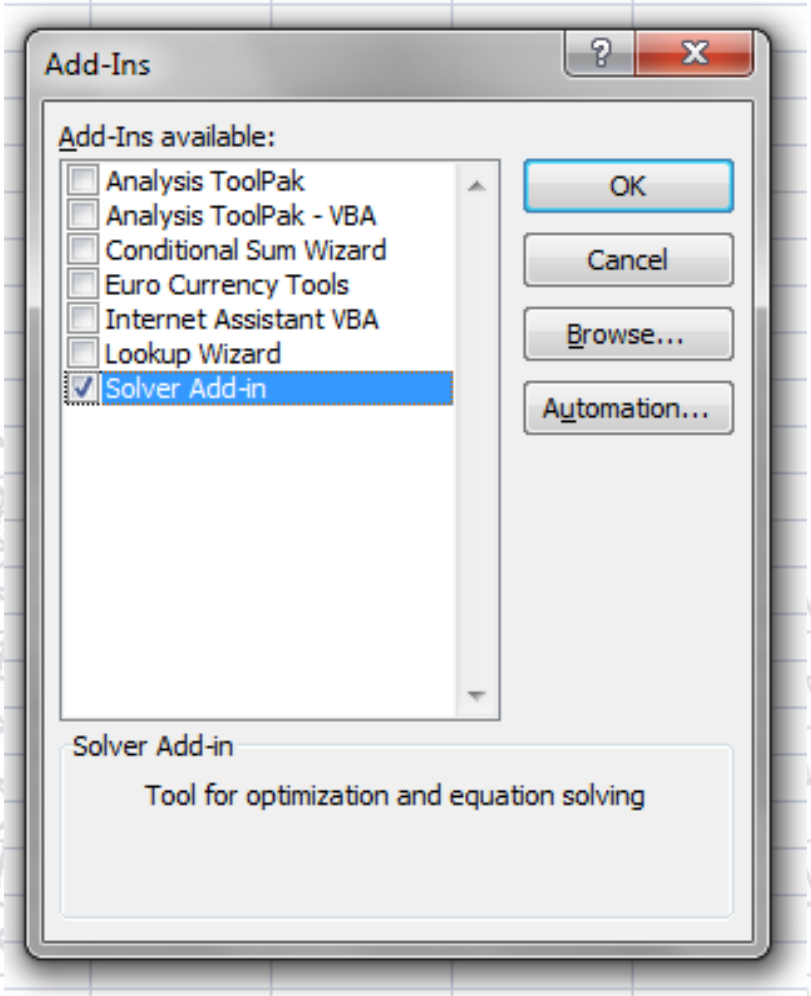
Type of Constraint	Symbol
Not greater than	≤
Not less than	≥
Equal to	=
Integer	int
Binary	bin
Non-negative	≥ 0

Problem	Constraints
Product mix	Product mix produced cannot use more resources than are available
Production planning	Do not produce products more than demand
Investment	Obtain an expected return of at least 10 percent on the investment

How to use Solver

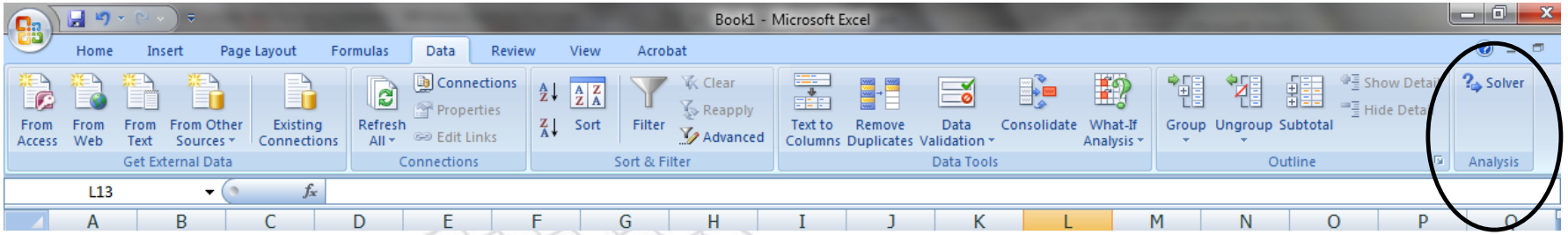
- Activate Solver: add-in that is not installed by default

File>Excel Option>Add-in>
Manage Excel Add-in>
Select the Solver Add-in checkbox



How to use Solver

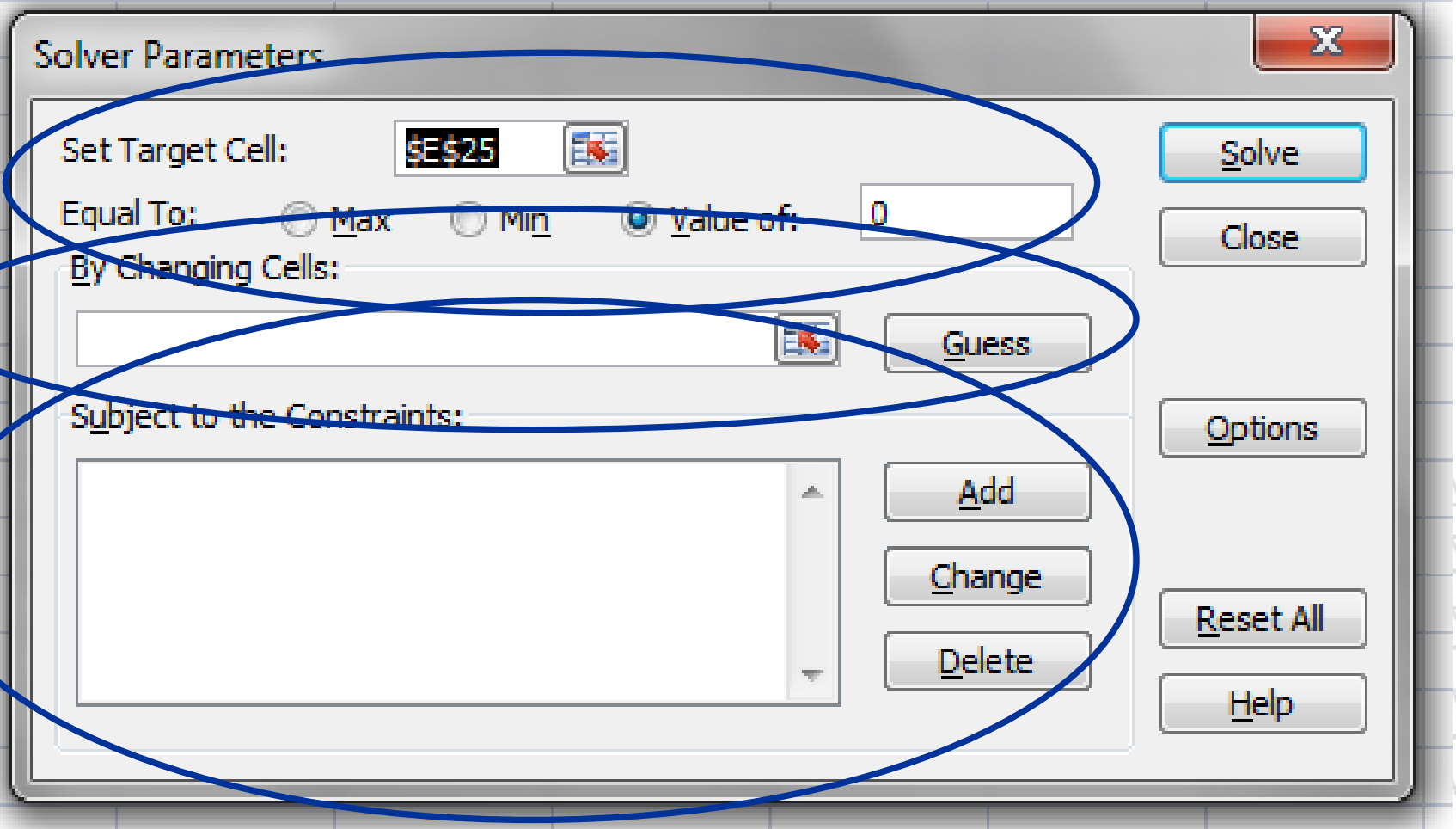
Solver will be available as a new Tab on the data, Analysis Tab



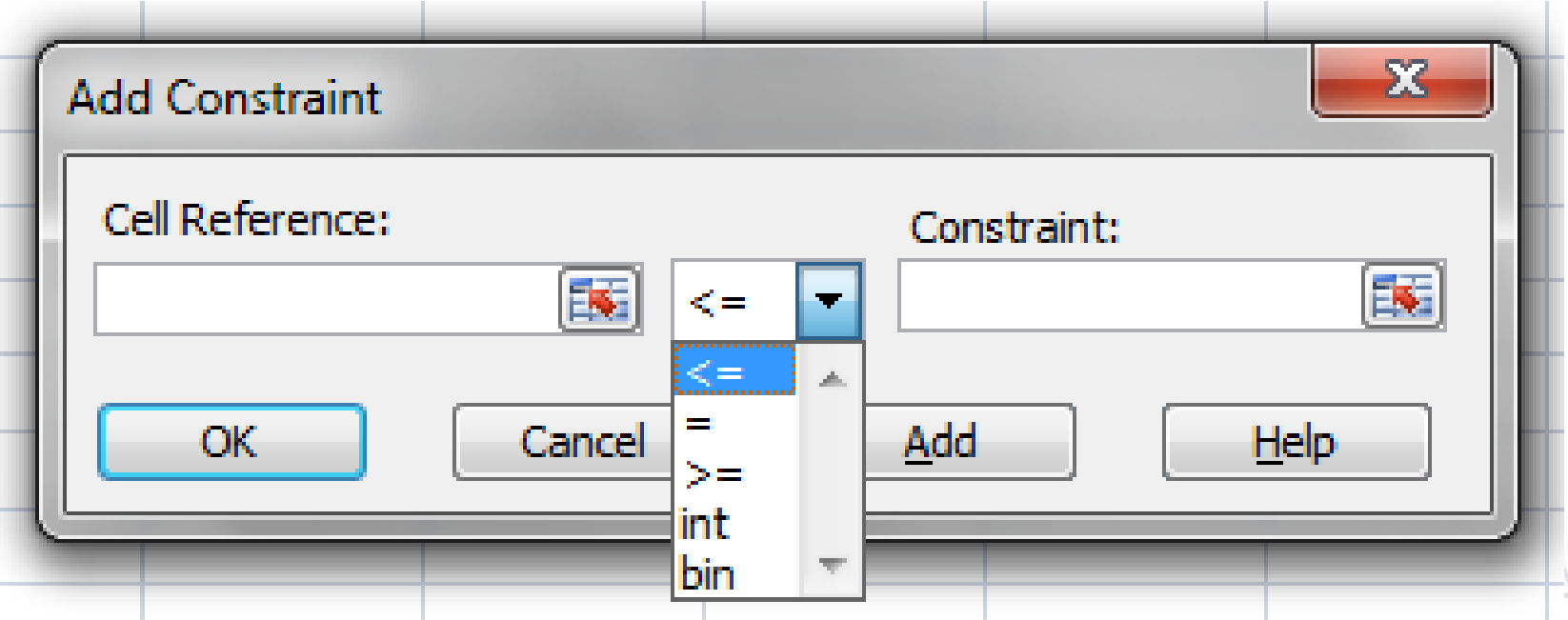
How to use Solver

- Determine target cell(s): must be a *Formula* cell containing changing cells
  - maximize, minimize, set value
- Determine changing cells: must 'feed' into target cell
- Establish constraints: the 'key' to make Solver work !!

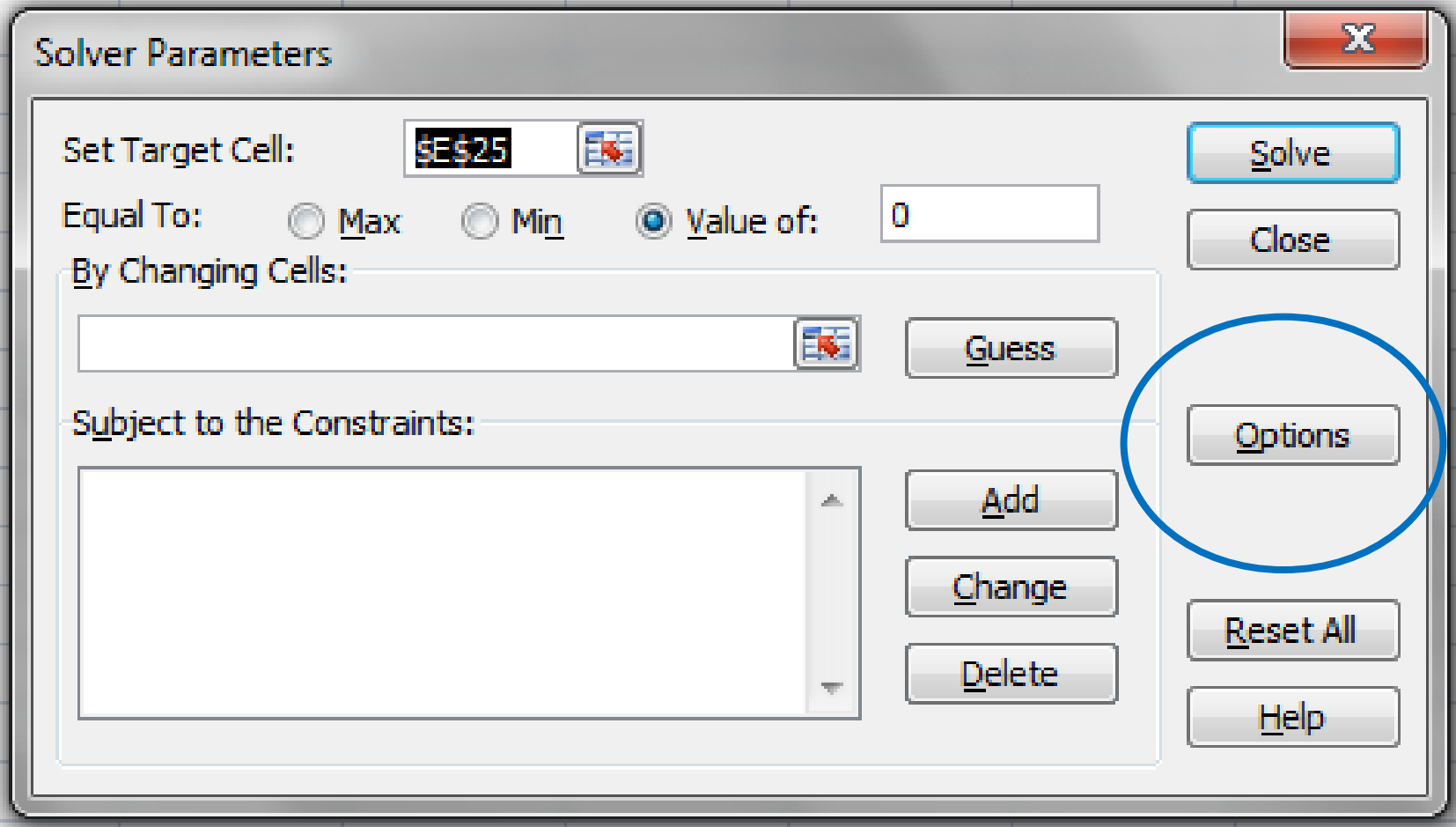
How to use Solver



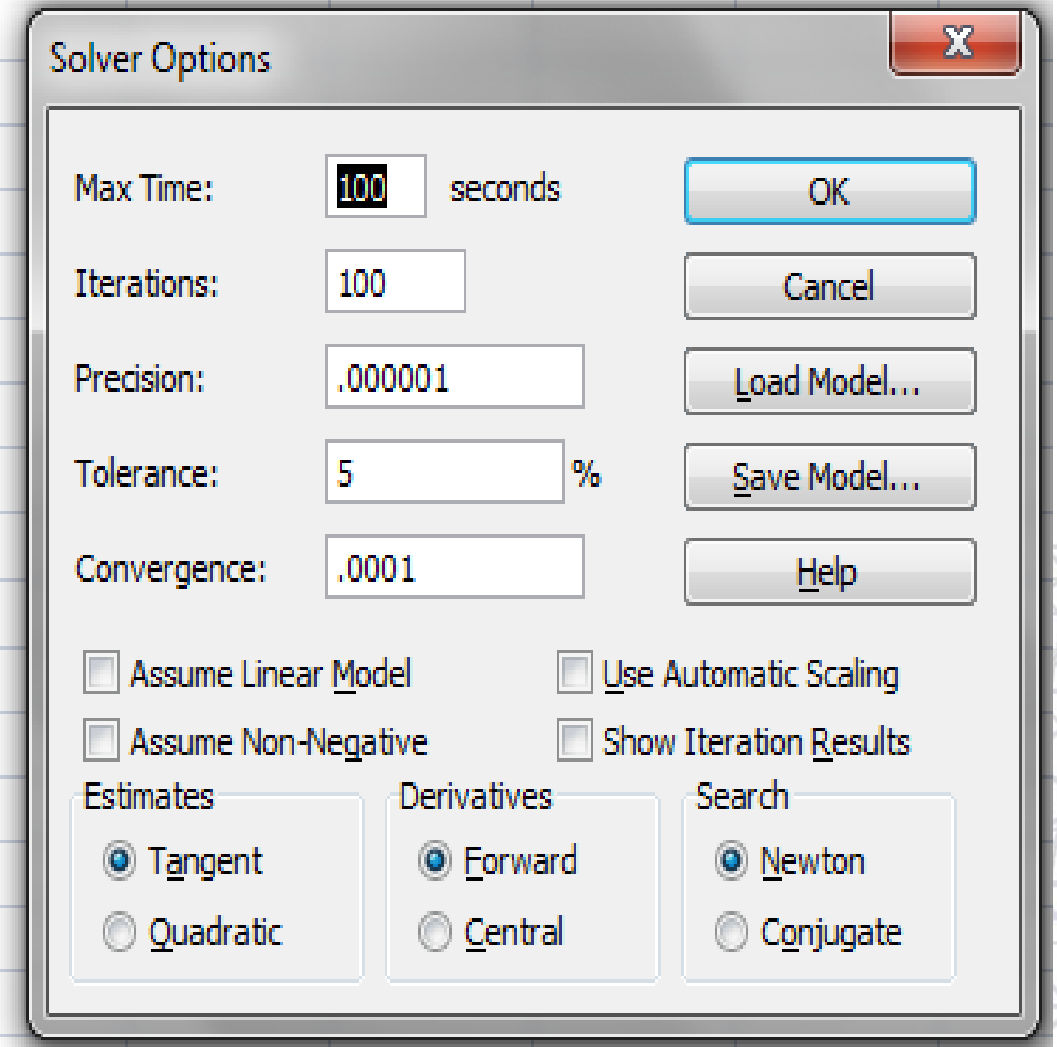
How to use Solver



How to use Solver



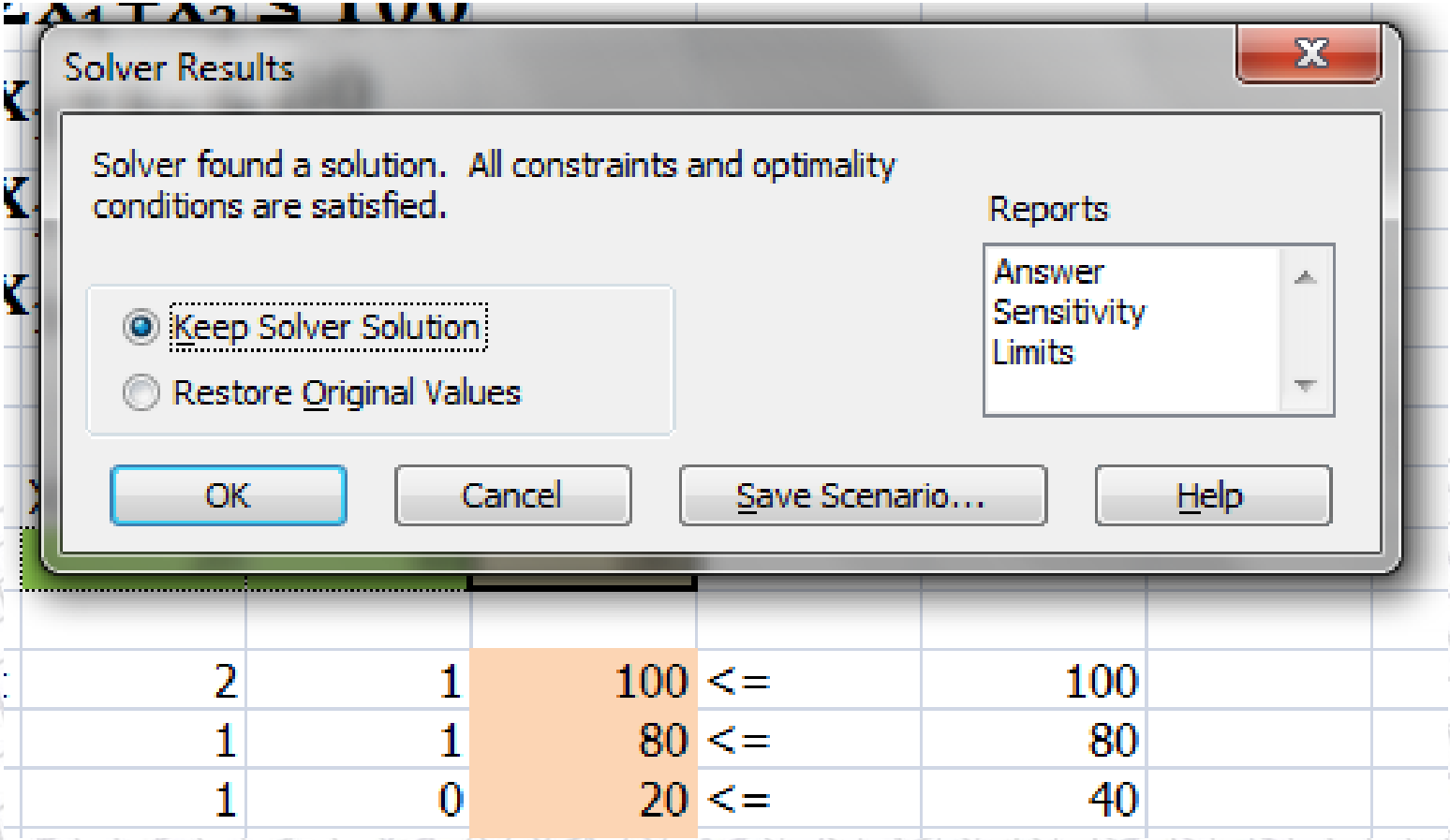
How to use Solver



Precision is the real number from 0 to 1 higher numbers means more precise

Tolerance shows how far away from the true optimal value and still be acceptable. Only applied to the problem with integer constraints

How to use Solver



Functions in Excel

- SUM()
- SUMPRODUCT()
  - Sum of product of 2 data sets [][] which are corresponding to each other
  - Example

	L	M	N
3			
4	3	4	2
5	1	5	0
6			

SUMPRODUCT(L4:N4,L5:N5)=(3*1)+(4*5)+(2*0)

- Etc….

Example

$$\text{Max } z = 3x_1+2x_2$$
$$\text{s.t.} \quad 2x_1+x_2 \leq 100$$
$$x_1+x_2 \leq 80$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

Product mix example

The XYZ company makes two products: Doors and Windows. Three manufacturing resources are required: Cutting, Sanding and Finishing.
The requirements measured in hours per unit and shown in the table below along with the profit per unit of product. The availability of Cutting, sanding , and finishing operations are 40, 40, and 60 hours respectively. Determine the quantity of doors and windows to be produced to maximize the profit of the company.

Product	Doors	Windows
Cutting Hours per Unit	1	0.5
Sanding Hours per Unit	0.5	0.75
Finishing Hours per Unit	0.5	1
Profit per Unit	$500	$400

Product mix example

Formulate the Product mix problem

X_1 : number of Doors produced

X_2 : number of Windows produced

z : company profit

$$\begin{aligned} \max\ z &= 500X_1 + 400X_2 \\ \text{s.t} \quad & X_1 + 0.5X_2 \leq 40 \\ & 0.5X_1 + 0.75X_2 \leq 40 \\ & 0.5X_1 + X_2 \leq 60 \\ & X_1, X_2 \geq 0 \text{ (non-negative constraint)} \end{aligned}$$

Diet Problem

My diet requires that all the food I eat come from one of the four "basic food groups"(chocolate cake, ice cream, soda, and cheesecake). Now, the following four foods are available for consumption: brownies, chocolate ice cream, cola, and pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must consume at least 500 calories, 6 oz of chocolate ,10 oz of sugar, and 8 oz of fat. The nutrition al content per unit of each food is shown in Table2. Solve this problem that can be used to satisfy my daily nutritional requirements at minimum cost.

Type of food	Calories	Chocolate (oz)	Sugar (oz)	Fat (oz)
Brownies	400	3	2	2
Chocolate ice cream (1 scoop)	200	2	2	4
Cola (1 bottle)	150	0	4	1
Pineapple Cheesecake (1 piece)	500	0	4	5

Transportation Problem

Powerco has 3 electric power plants that supply the needs of 4 cities. The costs of sending 1 million kwh of electricity from plant to city are shown in the table. Use Solver to minimize the cost of this problem.

	To				Supply
From	City1	City2	City3	City4	(million kwh)
Plant1	$8	$6	$10	$9	35
Plant2	$9	$12	$13	$7	50
Plant3	$14	$9	$16	$5	40
Demand (million kwh)	45	20	30	30	

Assignment Problem

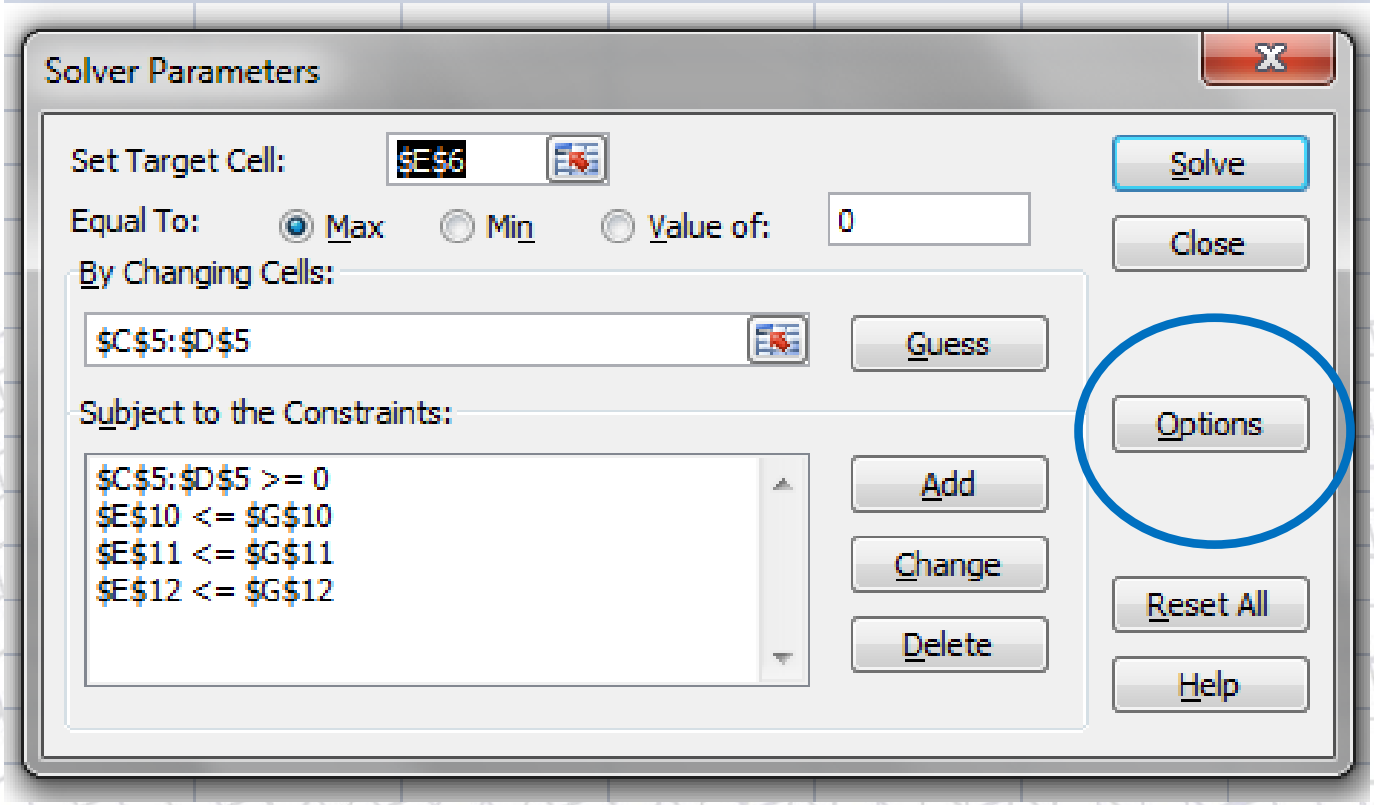
Machineco has 4 machines and 4 jobs to be completed. Each machine must be assigned to complete one job. The time required to set up each machine for completing each job is shown the Table. Use Solver to minimize the total set up time needed to complete the 4 jobs.

	Time (Hours)			
Machine	**Job 1**	**Job 2**	**Job 3**	**Job 4**
1	14	5	8	7
2	2	12	6	5
3	7	8	3	9
4	2	4	6	10

Step through Solver Trial Solutions

Solutions can be observed step by step:

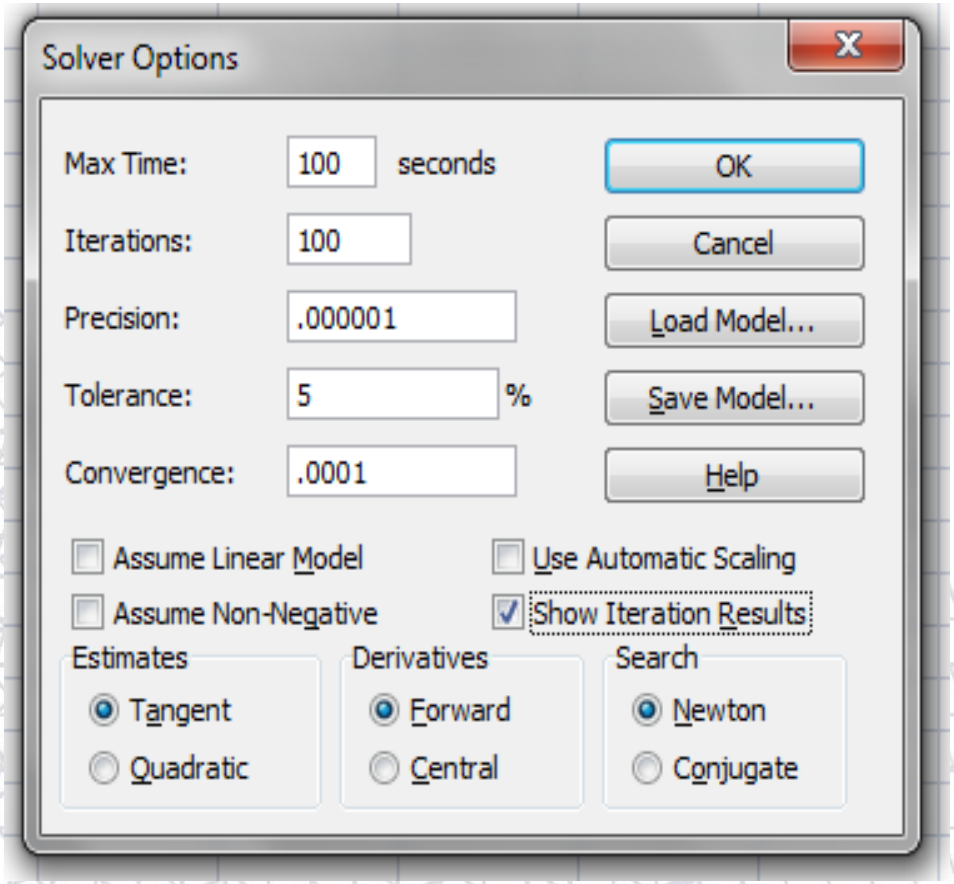
Solver Parameters>Options



Step through Solver Trial Solutions

Select the ***Show Iteration Results*** check box to see the values of each trial solutions --- > click ***OK***

During the run;
Stop: to stop the solution process
Continue: to continue the solution process



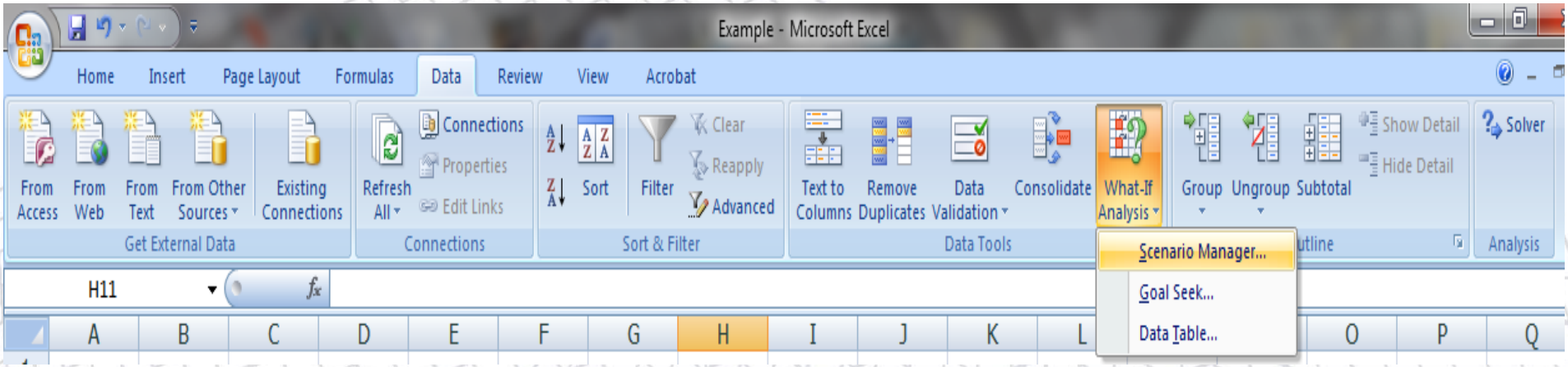
Solver Report

What-If Analysis

The process of changing the values in cells to see how those changes will effect the outcome of formulas on the worksheet

What-If Analysis tools in Excel:

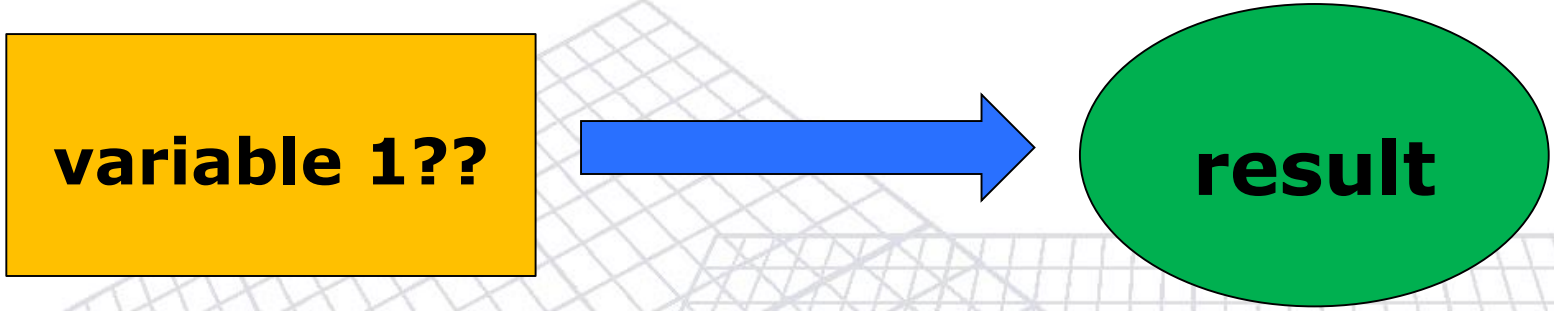
- **Scenarios** consider many different variables
- **Goal Seek** find out how to get a desired result
- **Data Tables** see the effects of one or two variables on formula



What-If Analysis

Goal Seek to find out how to get a desired result

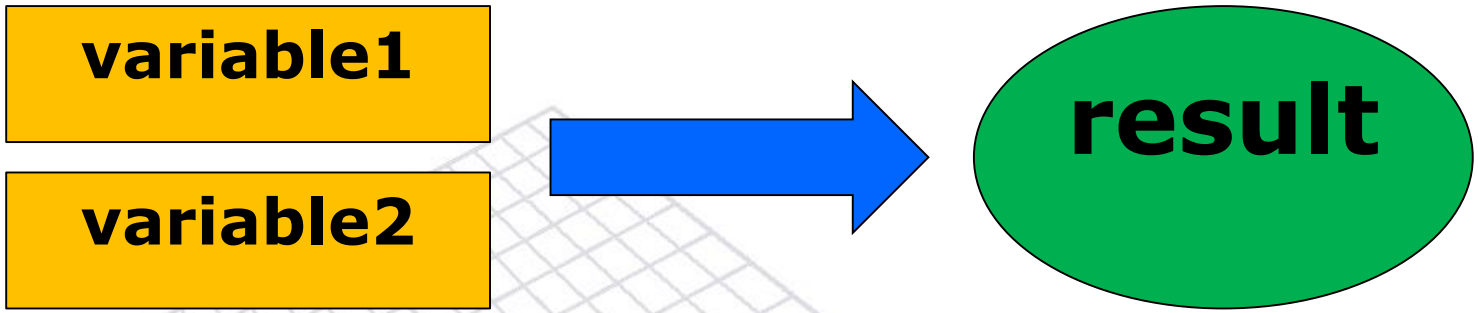
The target is to determine the possible input value that produce the result



Goal Seek work *with only one variable input* value!!

What-If Analysis

Data Tables see the effects of one or two variables on a formula/result

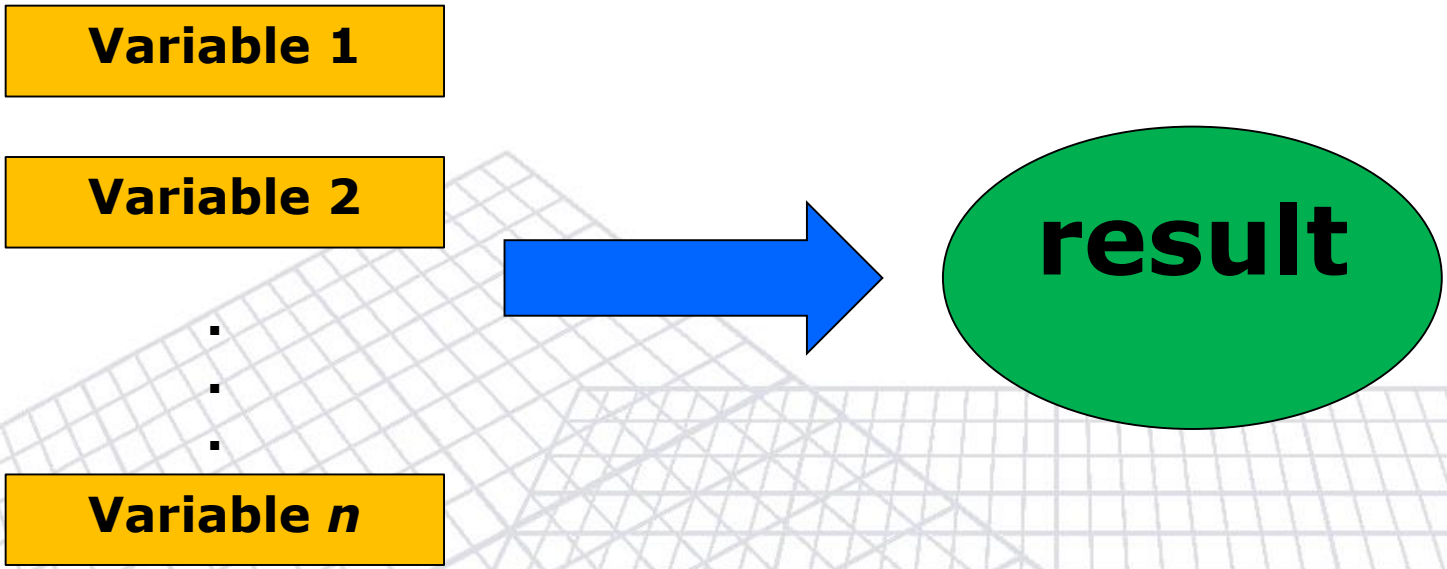


Data Table cannot work with more than two variables.
But it can take as many different variable values as wanted.

If a model has more than two variables, use scenarios.

What-If Analysis

Scenarios consider many different variables



It can take many input variables sets up to 32 different values
But as many scenarios as wanted can be created

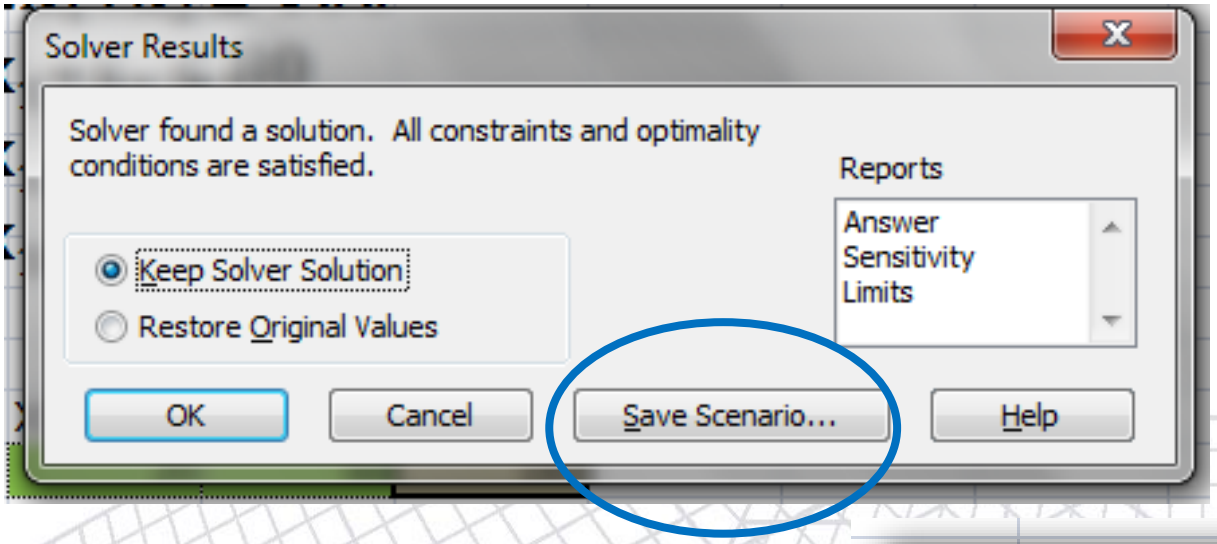
What-If Analysis

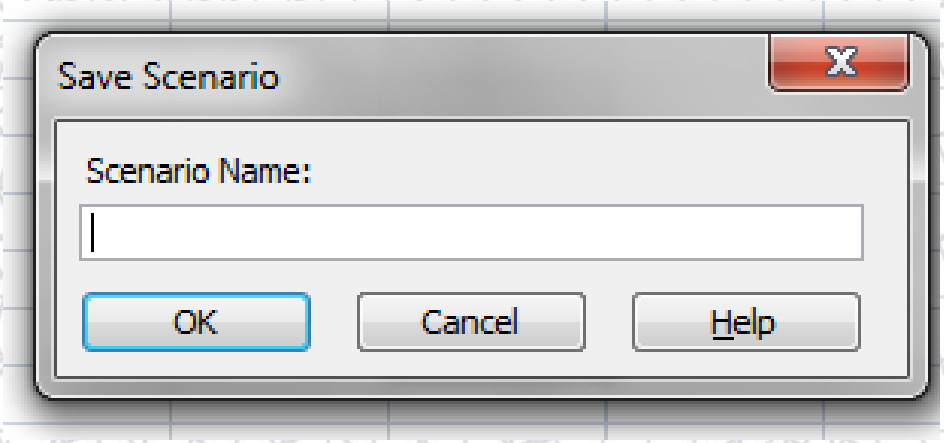
Solver is a tool for What-If Analysis

Solver is similar to Goal Seek except for the fact that it can determine more than one variable inputs

Solver Report

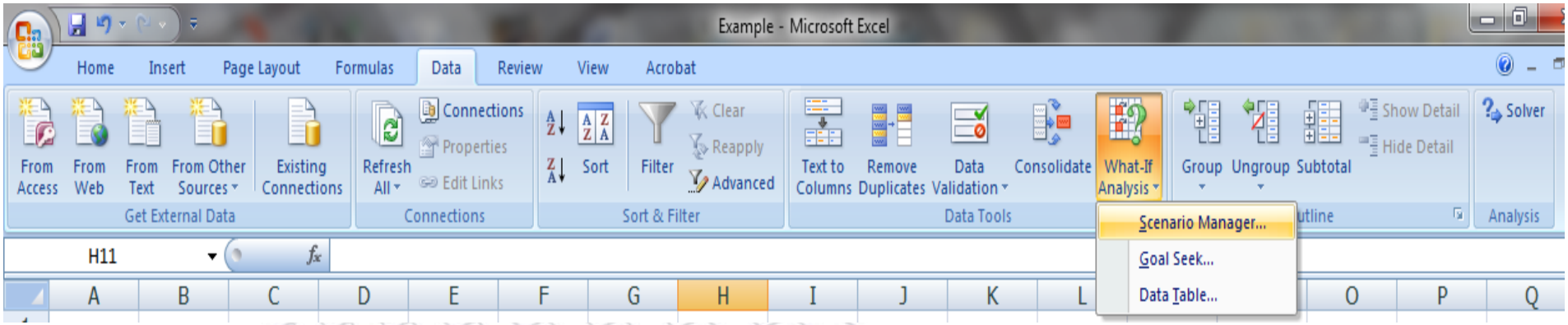
To save adjusting cell values as a scenario, click ***Save Scenario***, then type the name of the scenario in the ***Scenario Name*** box





Solver Report

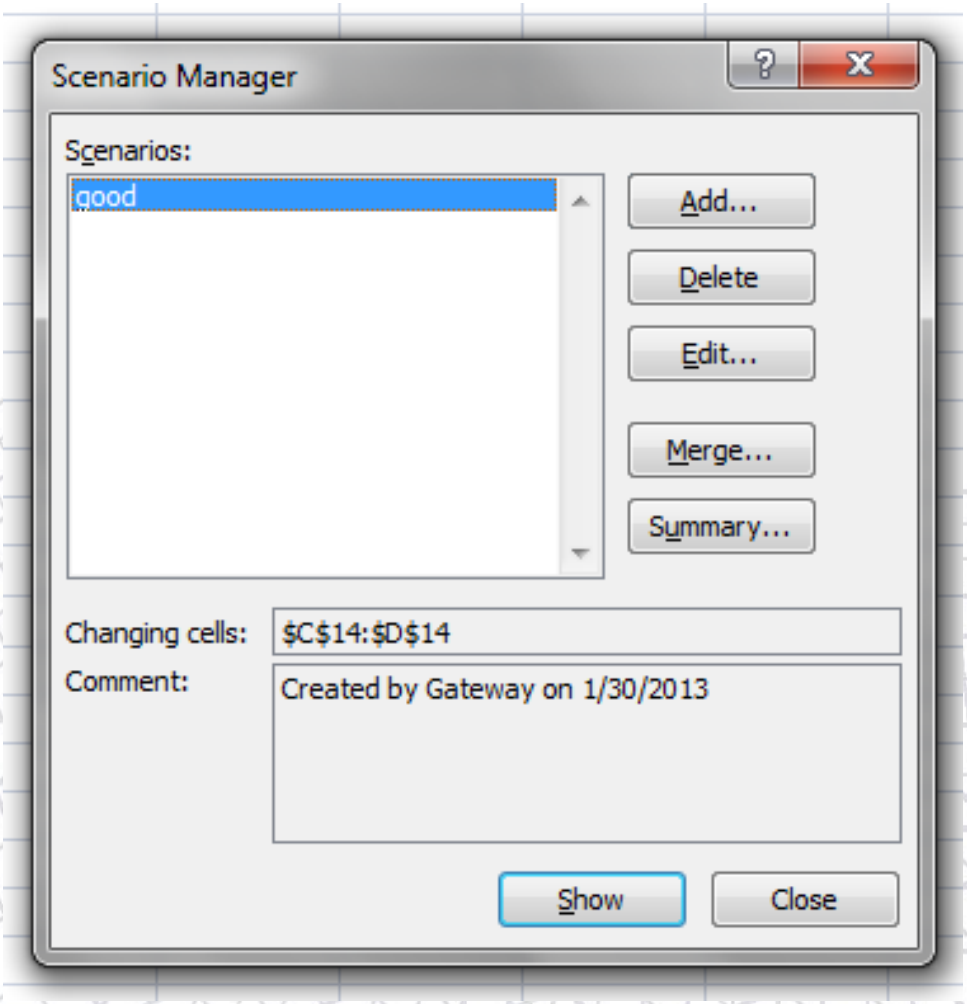
Scenario will be available for use in the Scenario Manager, which is accessed from ***Data*** > ***What-If-Analysis*** Tab



[See example]

Solver Report

Scenario Name is "**good**"



Solver Report

Scenario Name is "**good**".

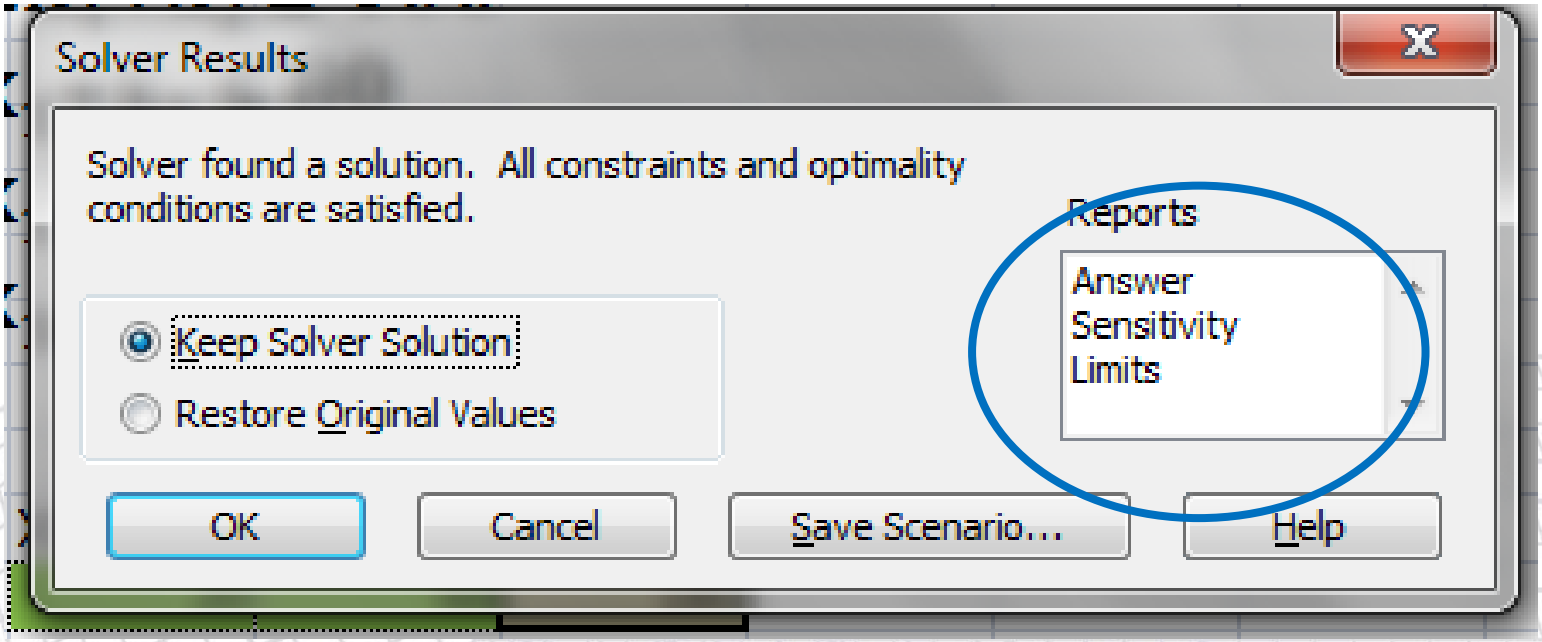
A	B	C	D	E	F	G

Scenario Summary		
	Current Values:	good
Changing Cells:		
$C$14	20	20
$D$14	60	60
Result Cells:		
$E$14	180	180
$E$16	100	100
$E$17	80	80
$E$18	20	20
$E$19	20	20
$E$20	60	60

Notes: Current Values column represents values of changing cells at time Scenario Summary Report was created. Changing cells for each scenario are highlighted in gray.

Solver Report

Report will be created on a *new worksheet* in the current workbook



Product mix example

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Finishing Hours per Unit	0.5	1
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Product mix example

	A	B	C	D	E	F	G	H	I
1	**Product Mix**								
2									
3									
4			Doors	Windows					
5		Qty	5	3					
6		Profit	500	400	3700				
7									
8									
9		Constraints	(all units in hours)		Used		Availability	slack	
10		Cutting	1	0.5	6.5	<=	40	33.5	
11		Sanding	0.5	0.75	4.75	<=	40	35.25	
12		Finishing	0.5	1	5.5	<=	60	54.5	
13									
14									

Answer Report

Answer Report reports

- The original and final objective values
- The original and final values of the variables
- The final values of constraints
- The constrains formulas, binding status and the slacks

Constraint with *no Slack* -→ ***Binding***

Answer Report

Microsoft Excel 12.0 Answer Report
Worksheet: [Example.xlsx]ProductMix
Report Created: 5/2/2556 10:35:23

Target Cell (Max)

Cell	Name	Original Value	Final Value
$E$6	Profit	3700	26000

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$5	Qty Doors	5	20
$D$5	Qty Windows	3	40

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$10	Cutting Used	40	$E$10<=$G$10	Binding	0
$E$11	Sanding Used	40	$E$11<=$G$11	Binding	0
$E$12	Finishing Used	50	$E$12<=$G$12	Not Binding	10
$C$5	Qty Doors	20	$C$5>=0	Not Binding	20
$D$5	Qty Windows	40	$D$5>=0	Not Binding	40

Constraint with *no Slack* -→ ***Binding***

Sensitivity Report

Sensitivity Report gives

- The optimal variable values
- The final values for constraints
- The *shadow prices* for the constraints with the maximum allowable increase and decrease
- The *reduced cost* for the input variables with the maximum allowable increase and decrease

Note: if the model contains integer or Boolean constraints, sensitivity report cannot be produced

Sensitivity Report

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [Example.xlsx]ProductMix
Report Created: 5/2/2556 13:29:23

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$5	Qty Doors	20	0	500	300	233.3333333
$D$5	Qty Windows	40	0	400	350	150

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$10	Cutting Used	40	350	40	40	13.33333333
$E$11	Sanding Used	40	300	40	6.666666667	20
$E$12	Finishing Used	50	0	60	1E+30	10

Sensitivity Report

Shadow Price/Dual price: determine how the objective values will change as you obtain an additional unit of constraints without re-run the Solver

If constraint is binding (no slack) --→ Shadow price ≠ 0
If constraint is no binding --→ Shadow price = 0

Sensitivity Report

Reduced Cost : determine additional cost/profit for every additional variable unit occurred

If reduced Cost ≤0 -→ Solution is optimal

Sensitivity Report

MAXIMIZATION (ex. Profit, Revenue)

If shadow price is positive → Profit increase
If shadow price is negative → Profit decrease

If reduced cost is positive → Profit increase
If reduced cost is negative → Profit decrease

MINIMIZATION (ex. Cost)

If shadow price is positive → Cost increase
If shadow price is negative → Cost decrease

If reduced cost is positive → Cost increase
If reduced cost is negative → Cost decrease

Sensitivity Report

Analysis: Product Mix Problem

1. If the availability of the cutting hours is increased to 80 hours, how much does the XYZ company gain their profit?

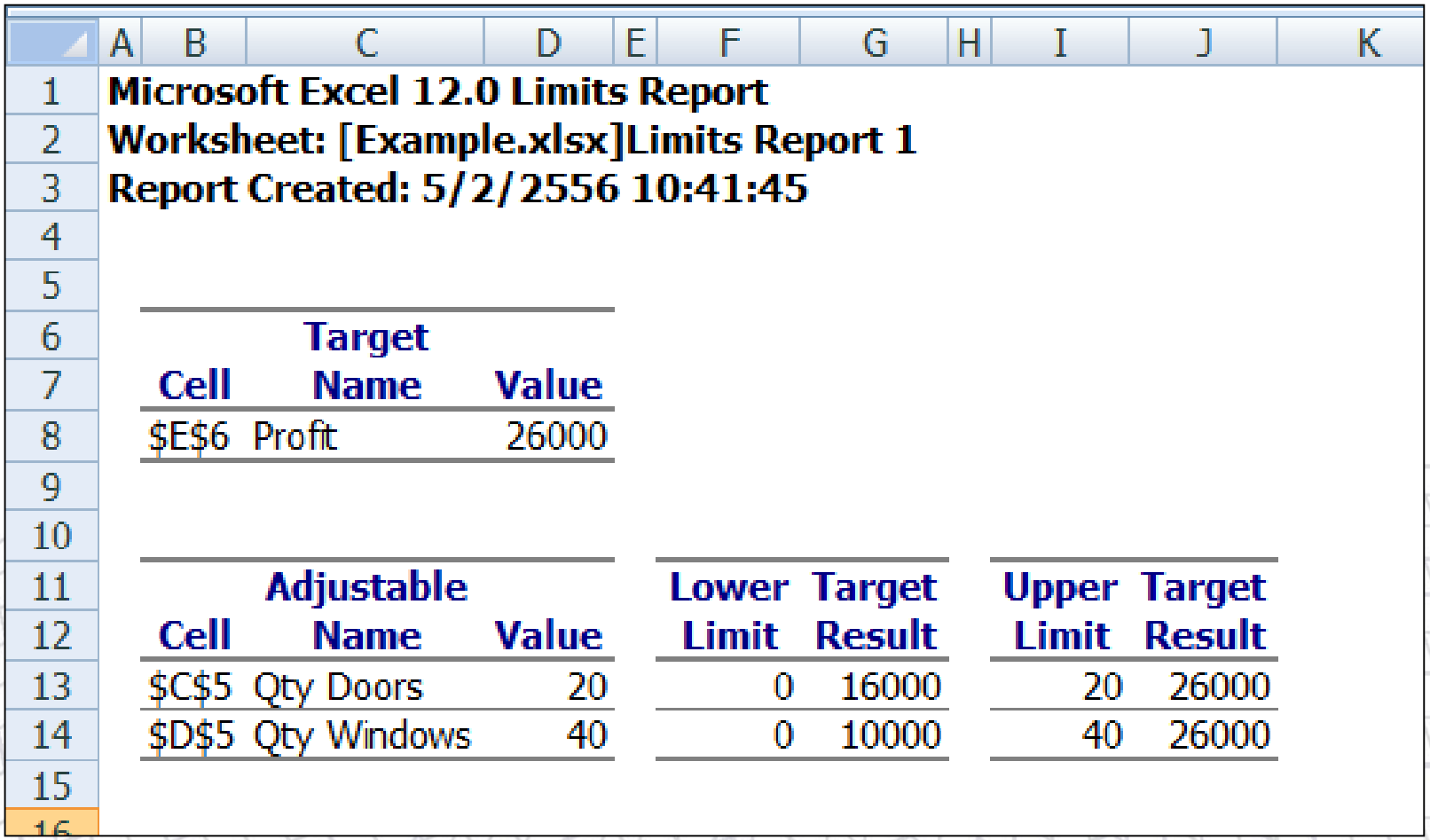
2. How much should the XYZ company pay for the sanding labor cost if the availability of sanding hours is increased to 41?

Limits Report

Limits Report reports

- The achieved optimal objective value
- The input variables with the optimal values and with lower and upper bound
- The lower bound indicates the smallest value that a variable can take while satisfying the constraints and holding all of the other variables constant
- The upper bound is the largest value the variable can take under these circumstances

Limits Report



Microsoft Excel 12.0 Limits Report
Worksheet: [Example.xlsx]Limits Report 1
Report Created: 5/2/2556 10:41:45

	Target		
Cell	Name	Value	
$E$6	Profit	26000	

	Adjustable		Lower	Target	Upper	Target
Cell	Name	Value	Limit	Result	Limit	Result
$C$5	Qty Doors	20	0	16000	20	26000
$D$5	Qty Windows	40	0	10000	40	26000

Diet Problem

My diet requires that all the food I eat come from one of the four "basic food groups"(chocolate cake, ice cream, soda, and cheesecake). Now, the following four foods are available for consumption: brownies, chocolate ice cream, cola, and pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must consume at least 500 calories, 6 oz of chocolate ,10 oz of sugar, and 8 oz of fat. The nutrition al content per unit of each food is shown in Table2. Solve this problem that can be used to satisfy my daily nutritional requirements at minimum cost.

Type of food	Calories	Chocolate (oz)	Sugar (oz)	Fat (oz)
Brownies	400	3	2	2
Chocolate ice cream (1 scoop)	200	2	2	4
Cola (1 bottle)	150	0	4	1
Pineapple Cheesecake (1 piece)	500	0	4	5

Diet Problem

Create sensitivity report and answer the following questions

1. If you must consume total calories at least 600 oz, what is the new cost?
2. If you must consume chocolate at least 8 oz, what is the new cost ?
3. If you want to eat 1 piece of brownie, what is the new cost?

Transportation Problem

Powerco has 3 electric power plants that supply the needs of 4 cities. The costs of sending 1 million kwh of electricity from plant to city are shown in the table. Use Solver to minimize the cost of this problem.

	To				Supply
From	City1	City2	City3	City4	(million kwh)
Plant1	$8	$6	$10	$9	35
Plant2	$9	$12	$13	$7	50
Plant3	$14	$9	$16	$5	40
Demand (million kwh)	45	20	30	30	

Transportation Problem

Create sensitivity report and answer the following questions

1. If electricity supply of Plant1 increases by 5 million kwh, how is it effect the cost? What will be the new cost?
2. If electricity supply of Plant2 increases by 5 million kwh, how is it effect the cost? What will be the new cost?

Inventory Problem

Sailco Corporation must determine how many sailboats should be produced during each of the next four quarters. The demand during each of the next four quarters is as follows: 40, 60, 75, and 25 sailboats. Sailco must meet demand on time. At the beginning of the first quarter, Sailco has an inventory of 10 sailboats. Sailco must decide how many sailboats should be produces during the quarter. For simplicity, we assume that sailboats manufactured during a quarter can be used to meet demand for that quarter. During each quarter, Sailco can produce up to 40 sailboats with regular time labor at a total cost of $400 per sailboats. By having employees work overtime during a quarter, Sailco can produce additional sailboats with overtime labor at a total cost of $450 per sailboats.

At the end of each quarter(after production has occurred and the current quarter's demand has been satisfied), a carrying or holding cost of $20 per sailboats is incurred. Determine production schedule to minimize the sum of production and inventory cost during the next four quarters.